

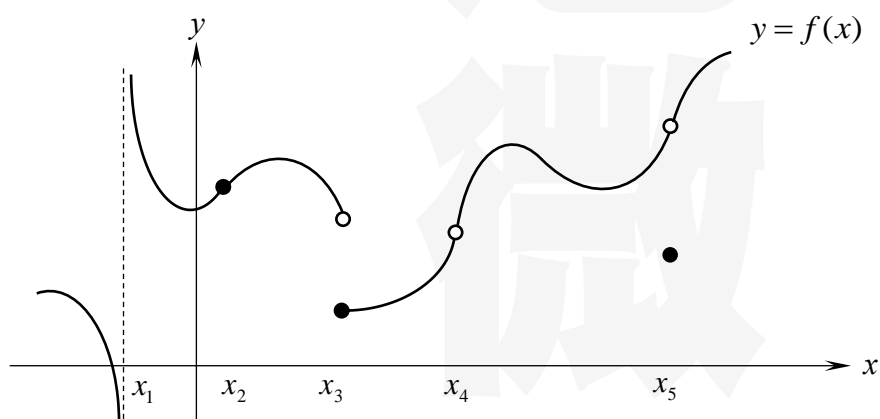
第一章 極限

- 今天的我，沒有極限

重點一 極限的直觀定義

1. 給定一個函數 $y = f(x)$ ，我們用

- (1) _____ 表示當 x 從左方往 x_0 靠近時， $f(x)$ 會靠近的值
- (2) _____ 表示當 x 從右方往 x_0 靠近時， $f(x)$ 會靠近的值
- (3) _____ 表示當 x 往 x_0 靠近時， $f(x)$ 會靠近的值



x_1 : _____

x_2 : _____

x_3 : _____

x_4 : _____

x_5 : _____

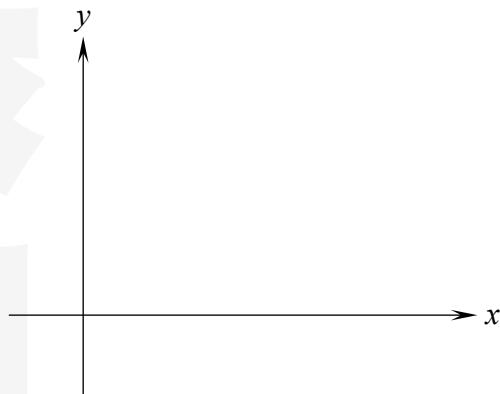
2. 極限存在的直觀定義：

$$\lim_{x \rightarrow x_0} f(x) = L \Leftrightarrow$$

例題 1.

Let $f(x) = 3$. Does $\lim_{x \rightarrow 5} f(x)$ exist? If it does, what is the value?

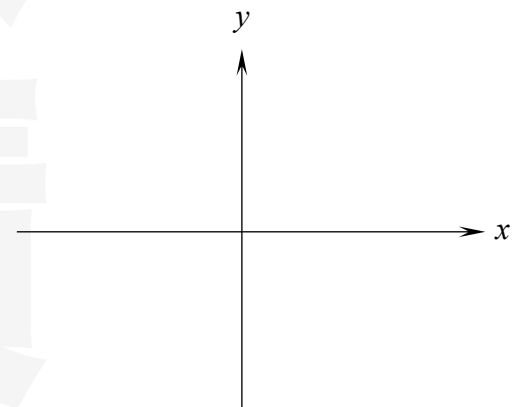
解



例題 2.

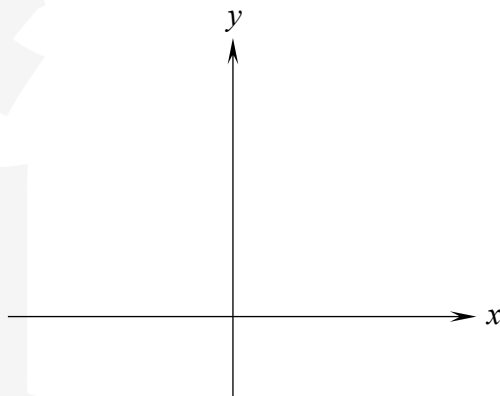
Let $f(x) = x$. Does $\lim_{x \rightarrow -1} f(x)$ exist? If it does, what is the value?

解



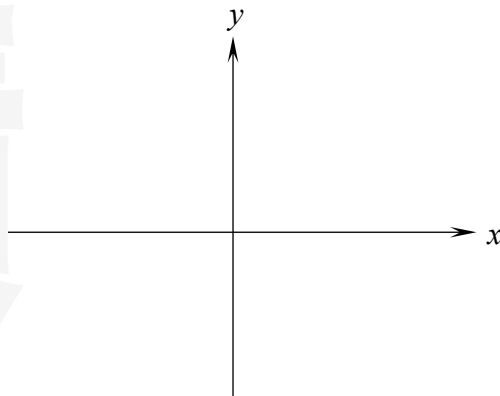
例題 3.

Let $f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$. Does $\lim_{x \rightarrow 0} f(x)$ exist? If it does, what is the value?

解

例題 4.

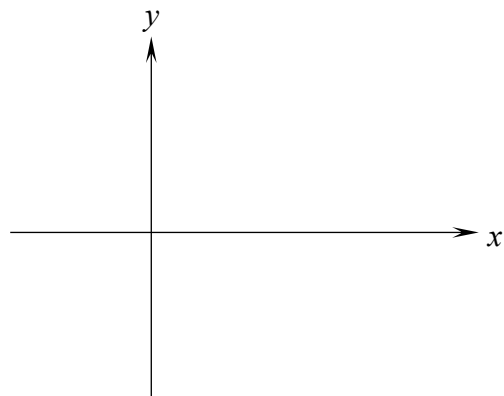
Let $f(x) = \begin{cases} -1 & \text{if } x > 0 \\ 1 & \text{if } x < 0 \end{cases}$. Does $\lim_{x \rightarrow 0} f(x)$ exist? If it does, what is the value?

解

例題 5.

Let $f(x) = \frac{1}{x-2}$. Does $\lim_{x \rightarrow 2} f(x)$ exist? If it does, what is the value?

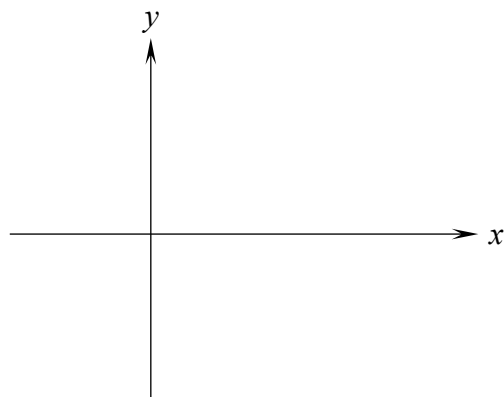
解



例題 6. (精選範例 1-1)

Let $f(x) = \begin{cases} -1 & \text{if } x \in \mathbb{Q} \\ 1 & \text{if } x \notin \mathbb{Q} \end{cases}$. Does $\lim_{x \rightarrow 0} f(x)$ exist? If it does, what is the value?

解



例題 7. (精選範例 1-2)

Let $f(x) = \sin(2x + \pi)$. Does $\lim_{x \rightarrow 0} f(x)$ exist? If it does, what is the value?

解

