習題集 $\boxed{1}$

(對應 張旭微積分 微分應用篇重點一：均值定理)

1. Show that $\left|\cos x - \cos y\right| \le \left|x - y\right|$ for all $x \in \mathbb{R}$ or all $x \in \mathbb{R}$.

2. Let $f(x) = x^4 - x$ be defined on $[0,1]$. Verify the Mean Value Theorem and find $c \in (0,1)$ that guaranteed by the theorem.

3. Suppose that $f(0) = -3$ and $f'(x) \le 5$ for all values of x. How large can $f(2)$ possibly be ?

4. Suppose that $f(x)$ is differentaible on $(0,1)$ and continuous on $[0,1]$. Given that $10 \ge f'(x) \ge 0$ for all $x \in (0,1)$, show that

$$10 + f(0) \ge f(1) \ge f(0).$$

5. Let $f(x) = 1 - \sqrt[3]{x^2}$. Show that $f(-1) = f(1)$. Is there any $c \in (-1,1)$ so that $f(1) - f(-1) = f'(c)(1-0)$. Does this fact contradict the Mean Value Theorem ?

6. At 2:00PM a car is at the position of 40KM on a highway. At 2:30 It arrives the position of 100KM. Does the ever exceed the speed limit of 100KM per hour at any moment ? How if the speed limit is 120KM per hour ?

7. Show that for $x \ge 0$, $x - \dfrac{x^3}{3!} \le \sin x \le x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$.

8. Show that for all $x \in \mathbb{R}$, $1 - \dfrac{x^2}{2!} \le \cos x \le 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!}$.

9. Show that the equation $x^3 + e^x = 0$ has exactly one real root.

10. If $f'(x) = 0$ for all $x \in (a,b)$ then $f(x)$ is constant on (a,b). If $f'(x) = g'(x)$ for all x in an interval (a,b), then $f(x) = g(x) + C$ for some constant C.