習題集 7

(對應 張旭微積分 微分應用篇重點七：微分量)

1. Use a differential (linearization) to estimate $\sqrt{104}$.
2. Use differentials to approximate $\sqrt{99.4}$.
3. Use differentials to approximate $\sqrt[3]{26}$.
4. Estimate $\sin 61^o$ using linearization method. [Knowing $\sqrt{3} \approx 1.732$ and $\pi \approx 3.14$]
5. Use a differential to estimate $\cos 40^o$.
6. Estimate the change in the potential energy of a rocket $U = -\dfrac{GM}{R}$ from a planet when the height changes from R_0 to $R_0 + \mathrm{d}R$. [Here G, M, R are the gravity constant, the mass of the planet, and the distance from rocket to the center of the planet, respectively]
7. The measurement of the edge of a cube is found to be 15 inches, with a possible error of 0.03 inch. Use differentials to approximate the maximum possible propagated error in computing (a) the volume of the cube and (b) the surface area of the cube.
8. The measurements of the base and altitude of a triangle are found to be 36 and 50 centimeters, respectively. The possible error in each measurement is 0.25 centimeter. Use differentials to approximate the possible propagated error in computing the area of the triangle.
9. The period P of the small oscillations of a simple pendulum is related to the length L of the pendulum by the equation

$$P = 2\pi\sqrt{\frac{L}{g}}$$

where g is the (constant) acceleration of gravity. Show that a small change $\mathrm{d}L\,\mathrm{dL}$ in the length of a pendulum produces a change $\mathrm{d}P$ in the period that satisfies the equation $\dfrac{\mathrm{d}P}{P} = \dfrac{\mathrm{d}L}{2L}$.

10. Let $f(x)$ be a function, $x_0 \in \mathbb{R}$, and

$$f(x) = f(x_0) + L(x - x_0) + \varepsilon_x (x - x_0).$$

Show that $f(x)$ is differentiable at $x = x_0$ with derivative L if and only if $\lim_{x \to x_0} \varepsilon_x = 0$.