習題集 3

(對應 張旭微積分 微分應用篇重點三：極值分析相關名詞介紹)

1. Show that $f(x)=-5x+1$ and $g(x)=\frac{1}{x}$ are strictly decreasing for $x>0$.
2. Show that $f(x)=\sqrt{x}$, $g(x)=x^3$ are strictly increasing.
3. Show that $f(x)=x^2-2x-2$ is increasing for $x\geq 3$.
4. Show that $f(x)=x^3+x$ is strictly increasing for $x\in\mathbb{R}$ and $g(x)=x^3-x$ is strictly increasing on $[3,\infty)$.
5. Show that $f(x)=\frac{1}{x}$ is decreasing for $x>0$.
6. Find critical points and inflaction points of the function $f(x)=x^3-3x$.
7. Show that $f(x)=\sin x$ and $g(x)=\tan x$ increase for $-\frac{\pi}{2}<x<\frac{\pi}{2}$. [Hint: Use the sum-to-product identities(和差化積公式) or Angle difference identities (和角公式). One can also consider $h(x)=\cos x$]
8. For $x\in[-1,1]$, is $f(x)=(\frac{1}{2})^{\cos\sqrt{1-x^2}}$ increasing or decreasing ?
9. Is it true that the sum of two increasing functions increasing ? How about product ?
10. Does every n-degree polynomial have $n-1$ critical points ?