

習題集 2

(對應 [張旭微積分](#) 微分應用篇重點二：微分與極限的聯手 (羅必達法則))

In question 1~5, find the following limits using L'Hôpital's rule.

1. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(66x)}{x}$.

2. Evaluate $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin(x + \frac{1}{4}\pi) - 1}{x - \frac{1}{4}\pi}$.

3. Let $f(x) = \begin{cases} \frac{\tan^{-1} x - x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Find $f'(0) + f(0)$.

4. Find $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{3!}x^3}{x^5}$.

5. Find $\lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{1}{2!}x^2}{x^4}$.

In question 6~8, find the following limits which are of the form 1^∞ , ∞^0 , or $0 \cdot \infty$.

6. Evaluate $\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}}$. Guess whether it equals to $\lim_{x \rightarrow \infty} (e^x + 0)^{\frac{1}{x}}$.

7. Let $\lambda \in \mathbb{R}$. Find $\lim_{x \rightarrow \infty} (1 + \frac{\lambda}{x})^{x+1}$.

8. Evaluate $\lim_{x \rightarrow 0^+} x \ln x$.

9. Show that if $f''(x_0)$ exists, then

$$f''(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + 2h) - 2f(x_0 + h) + f(x_0)}{h^2}.$$

10. Find $f'(0)$, $f''(0)$, and $f'''(0)$ of the function $f(x) = \begin{cases} e^{-\frac{1}{x}} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$ and

$$B'(1) \text{ of the Bump function } B(x) = \begin{cases} e^{1-x^2} & \text{if } |x| < 1. \\ 0 & \text{if } |x| \geq 1 \end{cases}.$$

$$[\text{Recall. } \frac{d}{dx} e^{-\frac{1}{x}} = \frac{1}{x^2} e^{-\frac{1}{x}} \text{ and } \frac{d^2}{dx^2} e^{-\frac{1}{x}} = e^{-\frac{1}{x}} \left(\frac{1}{x^4} - 2\frac{1}{x^3} \right)]$$