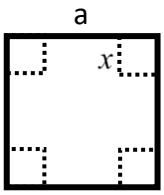
習題集 4

(對應 張旭微積分 微分應用篇重點四：微分求極值法)

1. Find all extrema: $f(x)=5x^2-10x+1$ on $[0,3]$.
2. Find all extrema: $f(x)=3\sin x-\cos x$.
3. Find all extrema: $f(x)=|x|^3-|x|$
4. Find all extrema: $f(x)=\left|x^3-x\right|$.
5. Find the nearest distance from $y=3x^2$ to $(0,3)$.
6. A box without a top is to be made by cutting small squares, of equal size from the corners of an 7×15 centimeter piece of card and then tuning up the sides. Find the maximum possible volume for the box.



7. Find all extrema: $f(x)=x^x$, $x>0$.
8. The combined resistance R of two resistors R_1,R_2 $(R_1,R_2>0)$ is given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$. Suppose R_1+R_2 is constant. Try to find the maximal combined resistance.
9. Two sources of heat are placed s meters apart—a source of intensity a at A and a source of intensity b at B. The intensity of heat at a point P on the line segment between A and B is given by the formula $I=\frac{a}{x^2}+\frac{b}{(s-x)^2}$, where x is the distance between P and A measured in meters. At what point between A and B will the temperature be lowest?
10. The figure shows a right circular cylinder inscribed in a sphere of radius r. Find the dimensions of the cylinder that maximize the volume of the cylinder.