習題集 1

(對應 張旭微積分 微分篇重點一：導數與微分的概念)

1. Find $f'(1)$ and $f'(x)$ if $f(x)=2x+3$.
2. Find $f'(1)$ and $f'(x)$ if $f(x)=x^3+3x$.
3. Find $f'(1)$ and $f'(x)$ if $f(x)=\begin{cases}\dfrac{x^2+1}{2} & \text{if } x>1\\ x & \text{if } x\le 1\end{cases}$.
4. Let $x_0>0$. Find $f'(x_0)+g'(x_0)$ if $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x}$.
5. Let $a,b>0$. If $f'(1)=6$, find $\lim\limits_{h\to 0}\dfrac{f(1+ah)-f(1-bh)}{h}$.
6. If $f(6)=f'(6)=6$, find $\lim\limits_{h\to 0}\dfrac{f(6+6h)-6}{h}$.
7. Let $f(x)=|x-1|+|x-2|$. Where is $f(x)$ non-differentiable ?
8. Let $f(x)=\begin{cases}x^2 & \text{if } x\in\mathbb{Q}\\ 0 & \text{if } x\notin\mathbb{Q}\end{cases}$. Where is $f(x)$ differentiable ?
9. Find $f'(1)$ and $f'(x)$ if $f(x)=(2x+1)^n$.
10. Find $f'(1)$ and $f'(x)$ if $f(x)=\sin(x^2)$.