

## 習題集 7

(對應 [張旭微積分](#) 微分篇重點七：微分工具整合)

1. Differentiate  $f(x) = \sin|x^2 + 2x + 2|$ .

2. Differentiate  $f(x) = 6^{\frac{6x + \sin x}{\sin 6x + 6}}$ .

3. Assume  $y = \sin(3u^2 - 5)$  and  $u = \log_3(5x - 2)$ . Find  $\frac{dy}{dx}$ .

4. If  $y = \frac{u^2 + 1}{u + 1}$ ,  $u = \log_2(3s)$ ,  $s = \sqrt[3]{t^2 + 1}$ , then  $\left. \frac{dy}{dt} \right|_{t=1} = ?$

5. Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for the equation  $x^2 + xy + 1 = y^2$ .

6. Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for the equation  $\frac{x - y}{x + y} = \frac{1}{x}$ .

7. Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for the equation  $\ln y = x^x$ .

8. Let  $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ .

(1) Find  $\lim_{x \rightarrow 0} f(x)$ .

(2) Find  $f'(0)$ .

(3) Find  $f'(x)$ .

(4) Is  $f'(x)$  continuous at  $x = 0$ ?

9. Find  $f'(0)$  if  $f(x) = \begin{cases} \frac{x^2}{\sqrt{1+x} - \sqrt{1-x}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ .

10. Find  $\left. \frac{dy}{dx} \right|_{(x,y)=(2,16)}$  and  $\left. \frac{d^2y}{dx^2} \right|_{(x,y)=(2,16)}$  for the equation  $\frac{\ln y}{\ln x} = x^x$ .