

習題集 4

(對應 [張旭微積分](#) 微分篇重點四：反三角函數的導函數)

1. Let $x_0 \in (0, \frac{\pi}{6})$. Find $(\sin^{-1} 3x + \cos^{-1} 3x)'$.
2. Find $[\sin^{-1}(\frac{x^2}{x^2+1})]'$.
3. Find $(\tan^{-1} \frac{x^2}{\sqrt{3}})'$.
4. Let $g(x) = \frac{6\sin^{-1} x - \pi}{2x-1}$. Find $g(\frac{1}{2})$ so that $g(x)$ is continuous
5. Find a, b for $f(x) = \begin{cases} \frac{\tan^{-1} 3x}{x} & \text{if } x < 0 \\ a - 5x & \text{if } 0 < x < 1 \\ \frac{b}{2} \cos(\sin(\pi x)) & \text{if } x \geq 1 \end{cases}$ to be continuous.
6. Find $[1.1x\sqrt{1-x^2} + \sin^{-1}(x)]'$.
7. Find $[1.1x\sin^{-1} x + \sqrt{1-x^2}]'$.
8. Show that for any $a \in [-1, 1]$, $\sin(\sin^{-1} a) = a$. How about $\sin^{-1}(\sin x)$ if $x \in \mathbb{R}$?
9. Show that when $x \in [-1, 1]$, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$. [Similar results for the other inverse trigonometric functions]
10. Let $f(x)$ be an strictly increasing function defined on $[a, b]$ that is invertible. If $f(x)$ is continuous, then $f^{-1}(y)$ is also continuous. If $f(x)$ is differentiable and $f'(x_0) \neq 0$, then $f^{-1}(y)$ is differentiable at $y = f(x_0)$. [本習題的主旨是要證明「連續函數的反函數也是連續的」、「可微函數的反函數也是可微的」的概念，但我們稍微拐個彎，這裡分成兩部份探討。另一部份「連續的一對一函數必是遞增或遞減」將放在[微分應用篇](#)的習題]