習題集 $\boxed{1}$

(對應 張旭微積分 極限篇主題一：極限的直觀定義)

1. Let $f(x) = 3x - 1$. Does $\lim_{x \to 0} f(x)$ exist? If it does, what is the value?

2. Let $f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$. Does $\lim_{x \to 0} f(x)$ exist? If it does, what is the value?

3. Let $f(x) = \dfrac{1}{x - \sqrt{5}}$. Does $\lim_{x \to 2} f(x)$ exist? Does $\lim_{x \to \sqrt{5}} f(x)$ exist? If any of them do, what is the value?

4. Let $f(x) = \dfrac{1}{x^2 - 4}$. Does $\lim_{x \to 2} f(x)$ exist? Does $\lim_{x \to -2} f(x)$ exist? If it does, what is the value?

5. Let $f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$. Does $\lim_{x \to 0} f(x)$ exist? Does $\lim_{x \to 0.01} f(x)$ exist? If any of them exist, what is the value?

6. Let $f(x) = \begin{cases} m & \text{if } x = m \in \mathbb{Z} \\ 0 & \text{if } x \notin \mathbb{Z} \end{cases}$. Find all points x_0 at which $\lim_{x \to x_0} f(x)$ fails to exist.

7. Let $f(x) = \begin{cases} \sin x & \text{if } x > 0 \\ \cos x & \text{if } x < 0 \end{cases}$ Does $\lim_{x \to 0} f(x)$ exist? If it does, what is the value?

8. Let $f(x) = \begin{cases} \sin x & \text{if } x > \dfrac{\pi}{4} \\ \cos x & \text{if } x < \dfrac{\pi}{4} \end{cases}$ Does $\lim_{x \to \frac{\pi}{4}} f(x)$ exist? If it does, what is the value?

9. Find $\lim_{x \to \pi} \cos(3x + \pi)$

10. Find $\lim_{x \to \sqrt{2}} \tan(\pi x^2)$