

習題集 9

(對應 [張旭微積分](#) 極限篇重點九：含無窮符號之極限)

1. Evaluate $\lim_{x \rightarrow -\infty} \left(\frac{1}{x^2} - \frac{1}{x} \right)$.
2. Evaluate $\lim_{x \rightarrow -\infty} \frac{3x^2 + 5}{x}$.
3. Evaluate $\lim_{x \rightarrow -\infty} \frac{3x^5 + 5}{x^5 + x^4 + x^3 + x^2 + x + 555}$.
4. Evaluate $\lim_{x \rightarrow -\infty} \log\left(\frac{1}{1 + \frac{1}{x}}\right)$.
5. Evaluate $\lim_{x \rightarrow -\infty} \log\left(\frac{x-1}{x+1}\right)$.
6. Show that if $\lim_{x \rightarrow \infty} f(x) = L$ and $\lim_{x \rightarrow \infty} g(x) = \infty$, then $\lim_{x \rightarrow \infty} f(x) + g(x) = \infty$.
[Similar rules about operations apply and one can try to establish the proofs]
7. Show that $\lim_{x \rightarrow 0^+} \ln x = -\infty$.
8. Show that $\lim_{x \rightarrow \infty} f(x) = L$ if and only if $\lim_{x \rightarrow 0^+} f\left(\frac{1}{x}\right) = L$. [This result will be used to show some case of L'Hôpital's Rule]
9. Show that $\lim_{x \rightarrow x_0} |f(x)| = \infty$ if and only if $\lim_{x \rightarrow x_0} \frac{1}{f(x)} = 0$.
10. Show that $\lim_{x \rightarrow \infty} \sin x$ doesn't exist.