習題集 5

(對應 張旭微積分 極限篇重點五：極限運算定理 (合成篇))

1. Evaluate $\lim_{x\to 0}\sqrt[3]{x^2+x+1}$.

2. Evaluate $\lim_{x\to 0}\dfrac{\sin(3^x+1)}{3^{3x}+1}$ and $\lim_{x\to 0}\sin^2(x^2+2x)$.

3. Evaluate $\lim_{x\to \frac{\pi}{2}}\log_{10}|\sin x|$.

4. Let p,q be polynomials. If $p(x_0)\neq 0$ and $\dfrac{q(x_0)}{p(x_0)}>0$, then

$$\lim_{x\to x_0}\sqrt[n]{\frac{q(x)}{p(x)}}=\sqrt[n]{\frac{q(x_0)}{p(x_0)}}.$$

5. Let $A\in\mathbb{R}$. If $\lim_{x\to 2}\dfrac{\sqrt{x^2+A}+2}{x-1}=6$, find A.

6. Let $f(x)=\begin{cases}3^{\sqrt[3]{x}}, & \text{if } x>0\\ 3^{\sqrt[3]{-x}}, & \text{if } x\le 0\end{cases}$. Find $\lim_{x\to 0}f(x)$. [Hint: One sided limits]

7. Let $f(x)=\begin{cases}x, & \text{if } x\neq 0\\ 1, & \text{if } x=0\end{cases}$ and $g(x)=\begin{cases}2^x, & \text{if } x\neq 1\\ \dfrac{1}{2}, & \text{if } x=1\end{cases}$. Find $\lim_{x\to 0}g(f(x))$.

8. Let $\lim_{x\to x_0}f(x)=L$ and $\lim_{t\to L}g(t)=M$. Does it necessarily hold that $\lim_{x\to x_0}g(f(x))=M$?

9. Evaluate $\lim_{x\to 1}x^x$.

10. Evaluate $\lim_{x\to 1}x^{x^x}$.