習題集 6

(對應 張旭微積分 極限篇重點六：去零因子求極限)

1. Evaluate $\lim\limits_{x \to 0} \dfrac{2x^3 - 2x^2 - 5x}{7x + 3x^2}$ and $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x^2 - 4}$

2. Evaluate $\lim\limits_{x \to 2}(\dfrac{1}{x} - \dfrac{1}{2})\dfrac{2x^2 - 3}{x - 2}$ and $\lim\limits_{x \to 0}(\dfrac{1}{x} - \dfrac{1}{2})\dfrac{2x^2 - 3}{x - 2}$.

3. Evaluate $\lim\limits_{x \to 0} \dfrac{(2x^2 - 3)\sqrt{1 - x}}{x - 1}$, and $\lim\limits_{x \to 0} \dfrac{\sqrt{1 - x} - \sqrt{1 + x}}{x(x + 1)}$.

4. Find $a + b + c$ if $\lim\limits_{x \to 1} \dfrac{\sqrt{x + a} - 2}{x - 1}$, $\lim\limits_{x \to 1} \dfrac{\sqrt{x + 5} - b}{x - 1}$, and $\lim\limits_{x \to c} \dfrac{\sqrt{x + 5} - 2}{x - c}$ all exist.

5. Find $\lim\limits_{x \to 1} \dfrac{1 - \sqrt[3]{x}}{1 - \sqrt{x}}$.

6. Evaluate $\lim\limits_{q \to 0} \dfrac{\sqrt[3]{q + \sqrt{4q^2 + 1}} + \sqrt[3]{q - \sqrt{4q^2 + 1}}}{q}$.

7. Evaluate $\lim\limits_{x \to 0} \dfrac{2^{2x} - 1}{2^x - 1}$, and $\lim\limits_{x \to 0} \dfrac{3^{3x} - 1}{3^x - 1}$.

8. Evaluate $\lim\limits_{x \to 2} \dfrac{\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + x}}}} - 2}{x - 2}$.

9. Find $\lim\limits_{y \to 0} \dfrac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4}$

10. Let $f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$. Let $a \neq 0$. Show that $\lim\limits_{x \to a} f(x)$ doesn't exist.

[這個敘述在連續篇的例題將當成已知]