

習題集 4

(對應 [張旭微積分](#) 極限篇重點四：極限運算定理 (四則運算篇))

1. Evaluate $\lim_{x \rightarrow 98} \frac{4-3x}{2+x}$ and $\lim_{x \rightarrow 1} \frac{x^5+x-2}{2+x^5}$.
2. Let $p(x)$ and $q(x)$ be polynomial functions and $p(x_0) \neq 0$. Show that
$$\lim_{x \rightarrow x_0} \frac{q(x)}{p(x)} = \frac{q(x_0)}{p(x_0)}.$$
3. Let $\lim_{x \rightarrow x_0} f_i(x) = a_i$, where a_i are real numbers and $n_i \in \mathbb{N}$. Show that
$$\lim_{x \rightarrow x_0} \sum_{i=1}^n c_i f_i(x)^{n_i} = \sum_{i=1}^n c_i a_i^{n_i}.$$
4. Evaluate $\lim_{x \rightarrow x_0} \sin\left(\frac{\pi}{3} + x\right)$, $\lim_{x \rightarrow x_0} \csc x$, and $\lim_{x \rightarrow \pi/3} \cos^n x$.
5. Evaluate $\lim_{x \rightarrow 1} \frac{x \sin x}{2+x^5}$, $\lim_{x \rightarrow \pi/4} \frac{\cos^3 x - x \sin x}{66x}$, and $\lim_{x \rightarrow \pi/4} \sin x \cos^2 x \tan^3 x$.
6. Evaluate $\lim_{x \rightarrow x_0} \frac{2^x + 2^{-x}}{2}$, $\lim_{x \rightarrow 0} \frac{2^{2x} - 2^{-2x}}{2^{2x} + 2^{-2x}}$, and $\lim_{x \rightarrow 1} \frac{x2^x - \sqrt{x}2^{2x}}{\sin x}$.
7. Suppose the limit $\lim_{x \rightarrow 2} f(x)$ exists and $\lim_{x \rightarrow 2} \frac{f(x)^4 + f(x) + 3}{f(x)^4 + 4} = 1$. Find $\lim_{x \rightarrow 2} f(x)$.
8. Show that if $\lim_{x \rightarrow x_0} f(x) = L$ then $\lim_{h \rightarrow 0} f(x_0 + h) = L$.
9. Show that if $\lim_{x \rightarrow x_0} |f(x)| = 0$ then $\lim_{x \rightarrow x_0} f(x) = 0$. How about the case that $\lim_{x \rightarrow x_0} |f(x)| = 2$?
10. Does it hold that $\lim_{x \rightarrow x_0} \sqrt{f(x)}$ exists $\Leftrightarrow \lim_{x \rightarrow x_0} f(x)$ exists ?