習題集 7

(對應 張旭微積分 積分前篇重點七：雙曲函數)

1. Show that $\cosh^2 x - \sinh^2 x = 1$.

2. Show that $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$

3. Show that $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

4. Show that $\sinh 2x = 2 \sinh x \cosh x$. How about $\cosh 2x$?

第 5~10 題皆會用到後面的概念，因此必須先學過後面的觀念再回來練習

5. Evaluate $\displaystyle\int_0^1 \sqrt{1+x^2}\, dx$. [本題用到變數變換法]

6. Evaluate $\displaystyle\int_1^2 \sqrt{x^2-1}\, dx$. [本題用到變數變換法]

7. Find the Taylor series of $f(x) = \sinh x$ at $x = 0$. [本題用到泰勒展開式]

8. Find the Taylor series of $f(x) = \cos x$ at $x = 0$. [本題用到泰勒展開式]

9. Find the Taylor series of $f(x) = \tanh^{-1} x$ at $x = 0$. [本題用到泰勒展開式]

10. If the colored area of the hyperbola $x^2 - y^2 = 1$ is A and the (x, y) and $(x, -y)$ are the coordinates of given vertices, show that $\dfrac{e^A + e^{-A}}{2} = \dfrac{x}{1}$.
The property gives another viewpoint of the defintion of the hyperbolic functions. [本題用到變數變換法]