

習題集 3

(對應 [張旭微積分](#) 積分前篇重點三：定積分正式定義)

本習題為體驗性質。讓同學感受到：若是僅僅憑定義，要證明給定的定積分值是相當費功的，如此一來，主題四、主題五所提供的工具就相當有價值了！

1. Show that $\int_{-1}^1 5 \, dx = 10$ by definition.
2. Show that $\int_{-1}^1 [x] \, dx = -1$ by definition.
3. Show that $\int_{-1}^1 3x+5 \, dx = 10$ by definition.
4. Show that $\int_{-1}^1 |x| \, dx = 1$ by definition.
5. Show that $\int_{-1}^1 5 - 3x \, dx = 10$ by definition.
6. Show that $\int_0^1 x^2 \, dx = \frac{1}{3}$ by definition.
7. Show that $\int_0^1 x + x^2 \, dx = \frac{5}{6}$ by definition.
8. Is $f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$ integrable over $[0,1]$?
9. Is $f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$ integrable over $[0,1]$?
10. Is the following Thomae's function integrable over $[0,1]$:
$$f(x) = \begin{cases} \frac{1}{p} & \text{if } x = \frac{q}{p} \text{ for some } p, q \in \mathbb{N} \text{ with } (p, q) = 1 \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases} ?$$