習題集 4

(對應 張旭微積分 積分前篇重點四：積分運算性質)

我們已經由正式定義確認了： $\int_a^b \mathrm{d}x = b-a$ 、$\int_a^b x^k \,\mathrm{d}x = \frac{x^b - x^a}{k+1}\,(k=1,2)$。也知道了 $\int_a^b [x]\,\mathrm{d}x$ 和 $\int_a^b |x|\,\mathrm{d}x$ 跟直觀做法相符。

1. Evaluate $\int_1^2 x+5\,\mathrm{d}x$.
2. Evaluate $\int_1^2 x+3[x]\,\mathrm{d}x$.
3. Evaluate $\int_1^2 x^2+x+1\,\mathrm{d}x$
4. Evaluate $\int_{-1}^1 |x|\,\mathrm{d}x$.
5. Suppose that $\int_1^4 f(x)\,\mathrm{d}x = 1$, $\int_3^4 f(x)\,\mathrm{d}x = 2$, $\int_3^7 f(x)\,\mathrm{d}x = 3$, $\int_5^7 f(x)\,\mathrm{d}x = 4$, $\int_5^6 f(x)\,\mathrm{d}x = 5$. Find $\int_1^6 f(x)\,\mathrm{d}x$, $\int_1^7 f(x)\,\mathrm{d}x$, and $\int_6^7 f(x)\,\mathrm{d}x$.
6. Find a function $f(x)$ that obeys the above conditions.
7. Suppose that $\int_1^2 f(x)\,\mathrm{d}x = 1$, $\int_2^7 f(x)\,\mathrm{d}x = 2$, $\int_5^7 f(x)\,\mathrm{d}x = 3$, $\int_6^7 f(x)\,\mathrm{d}x = 4$, $\int_5^4 f(x)\,\mathrm{d}x = 5$. Find $\int_1^4 f(x)\,\mathrm{d}x$, $\int_1^6 f(x)\,\mathrm{d}x$, and $\int_4^6 f(x)\,\mathrm{d}x$.
8. Find a function $f(x)$ that obeys the above conditions.

9. Show that If f is non-integrable and g is integrable then $f+g$ is non-integrable. Is it true that if f and g are non-integrable then $f+g$ is non-integrable as well ? [可以發現： 不可積函數並不會比可積函數要少]

10. Estimate $\int_0^{\pi} \sin x \, dx$ via the polygonal functions, where edges the outer trapezoid is given by the tangent of $y=\sin x$ at $(0,0)$, $(\frac{\pi}{2},1)$, $(\pi,0)$, and the base line, while the vertices of the inner pentagon are $(\frac{k\pi}{4},\sin\frac{k\pi}{4})$ for $k=0,1,2,3,4$.