習題集 $\boxed{1}$

(對應 張旭微積分 連續篇重點一：連續的概念)

1. Show that $f(x) = \dfrac{1}{x}$ is continuous everywhere except at $x = 0$ and that $g(x) = x^n$ is continuous everywhere for all $n \in \mathbb{N}$.

2. Show that $f(x) = \sqrt[n]{x^m}$ is continuous on $x > 0$ for any $m, n \in \mathbb{N}$.

3. Show that $f(x) = a^x$ is continuous eveywhere and that $f(x) = \log_a x$ for any $x > 0$. Here $a > 0$ is a real number.

4. Show that $f(x) = x^p$ is continuous on $x > 0$ for all $p \in \mathbb{R}$, $p \neq 0$.

5. Let $f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$. Where is $f(x)$ continuous ?

6. Let $f(x) = \begin{cases} 3x - 2 & \text{if } x \in \mathbb{Q} \\ x + 3 & \text{if } x \notin \mathbb{Q} \end{cases}$. Where is $f(x)$ continuous ?

7. Let $f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$. Where is $f(x)$ continuous ?

8. Let $a > 0$ and let $f(x) = \left| a - x^2 \right|$. Show that $f(x)$ is continuous everywhere.

9. Let $f(x) = [x]$. Where is $f(x)$ continuous ?

10. Let $f(x)$ be a function defined on $[-1, 1]$ that satisfies $x^2 + (f(x))^2 = 1$. Is $f(x)$ necessarily continuous ?