習題集 5

(對應 張旭微積分 連續篇重點五：極值定理)

本習題前五題爲練習性質，主要期許各位同學能夠熟悉極值定理的定理條件、使用方法；後五題則是體驗性質，讓同學瞭解在尚未學習微分工具之下，我們能夠處理函數極值問題的界限，藉此在 微分應用篇 學到統一的方法時，能感受到它的力量。

1. Find the maximum and the minimum of $f(x)=|x^2-2x-3|$ on $[0,4]$.

2. Show that the function $f(x)=\tan(\frac{x^2+x-1}{3}\pi)$ admits a maximum when $0\le x\le 1$.

3. Let $f(x)=\begin{cases} x & \text{if } x\neq 0 \\ 1 & \text{if } x=0 \end{cases}$, $x\in[-\frac{1}{2},\frac{1}{2}]$. Does $f(x)$ satisfy the conditions of the Extreme Value Theorem ? Does it have any extreme value ?

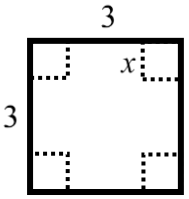
4. Show that $f(x)=2x^2-4x+3-(x^2-2x)^2$, $x\in[-3,3]$, has a maximum.

5. Prove that when $x\in\mathbb{R}$. The function $f(x)=\frac{x+1}{x^2+x+6}$ is bounded while $g(x)=\frac{x+1}{x^2+x-6}$ is not.

6. For $x>0$, Prove that the function $f(x)=x+\frac{3}{x+1}$ admits a minimum.

7. Prove that the function $f(x)=\frac{x+1}{x^2+x+6}$ admits a maximum and minimum for x ranging over the whole real number line.

8. A rectangle box is formed by cutting four equal corners from a square of side 10 and then folding up (see the figure). Find the maximum possible volume of the box.



9. Show that $f(x)=\sin x+\cos x$ has a maximum.

10. Show that $f(x)=x+2\sqrt{1-x^2}$ has a maximum and a minimum.