

習題集 4

(對應 [張旭微積分](#) 連續篇重點四：中間值定理)

1. Let $f(x) = \begin{cases} -x^2 - x & \text{if } x < 1 \\ x - 3 & \text{if } x \geq 1 \end{cases}$. Verify that the Intermediate Value Theorem applies to the interval $[-2, 3]$ and find the value of c so that $f(c) = -1$.
2. For $f(x) = x^3 - 2x^2 + 2$ on $[-2, 2]$ and $f(c) = 1$, verify that the Intermediate Value Theorem applies to the indicated interval and find the value of c guaranteed by the theorem.
3. Show that the equation $\sin x = 2^x - 1$ has a real root.
4. Show that the equation $\sin x = 2^x - 1$ has a real root between -3 and -1 . ($\pi \approx 3.14$)
5. Let $f(x) = \begin{cases} -|x| & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$. Does $f(x)$ satisfy the Intermediate Value Theorem? Why?
6. Find the root of $x^3 + x = 1$ that is accurate to 1 decimal place.
7. Let $f(x) = \frac{1}{x-1} + \frac{1}{x-4}$. Show that there is a number $c \in (1, 4)$ such that $f(c) = 0$. [[在微分應用篇](#)將會教此類函數的繪圖]
8. Suppose that the temperature on the earth's surface varies continuous with position. Prove that at any moment, there is always a place where the temperature is the same as its diametrically opposite place.
9. Can a continuous function $f(x)$ defined on $(0, 1)$ that takes on only three distinct values exist?
10. Show that any polynomial with odd degree admits a real root.