

重點五 極限運算定理 (合成篇)

◎ 若 $\lim_{x \rightarrow x_0} f(x) = L$ 且 $\lim_{t \rightarrow L} g(t) = g(L)$,

則 $\lim_{x \rightarrow x_0} g(f(x)) =$ _____

說明

1° Given $\varepsilon > 0$, since $\lim_{t \rightarrow L} g(t) = g(L)$

$\exists \delta_1 > 0$ such that, $\forall 0 < |t - L| < \delta_1$, $|g(t) - g(L)| < \varepsilon$.

$\because \lim_{x \rightarrow x_0} f(x) = L$,

$\therefore \exists \delta > 0$ such that, $\forall 0 < |x - x_0| < \delta$, $|f(x) - L| < \delta_1$.

2° Now, for such $\delta > 0$, if $0 < |x - x_0| < \delta$,

then $|f(x) - L| < \delta_1$ and thus $|g(f(x)) - g(L)| < \varepsilon$.

3° Since ε is arbitrary, $\lim_{x \rightarrow x_0} g(f(x)) = g(L)$. [Q.E.D.]

例題 1.

Find the following limits.

(1) $\lim_{x \rightarrow 1} \sqrt{x^3 + 1}$

(2) $\lim_{x \rightarrow 1} |x^2 + 2|$

解