重點七 微分工具整合

求導函數四大工具：

(1) 微分表

(2) 微分運算律、微分合成律

(3) 反函數微分法、隱函數微分法

(4) 微分定義式

例題 1. (精選範例 7-1)

Differentiate the following functions.

(1) $f(x) = \cos|2x+5|$

(2) $f(x) = 10^{\frac{1+x}{1-x}}$

解

例題 2. (精選範例 7-2)

(1) Assume that $y = \log_2 u$ and $u = 3x^4 + 5$. Find $\dfrac{dy}{dx}$.

(2) If $y = \dfrac{u+2}{u-1}$, $u = (3s-1)^{\frac{2}{3}}$, and $s = \sqrt{t}$, then $\left.\dfrac{dy}{dt}\right|_{t=9} = ?$

解

例題 3. (精選範例 7-3)

Find $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$ for the following equations.

(1) $x^2 - xy + y^2 = 1$ (2) $\dfrac{x+y}{x-y} = \dfrac{1}{y}$ (3) $\ln y = (2x+3)^x$

解

例題 4. (精選範例 7-4)

Let $f(x) = \begin{cases} x\sin\dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

(1) Calculate $\displaystyle\lim_{x\to 0} f(x)$ (2) Calculate $f'(0)$ (3) Find $f'(x)$

(4) Is $f'(x)$ continuous at $x = 0$?

解

例題 5. (精選範例 7-5)

Let $f(x) = \begin{cases} \dfrac{1-\cos x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Find $f'(0)$.

解