重點四 微積分基本定理 II - 先積再微型

設 $f(t)$ 爲連續函數。

◎ $\dfrac{d}{dx}\int_a^x f(t)dt =$ ________________

(1) $\dfrac{d}{dx}\int_x^a f(t)dt =$ ________________

(2) $\dfrac{d}{dx}\int_a^{g(x)} f(t)dt =$ ________________

(3) $\dfrac{d}{dx}\int_{g_1(x)}^{g_2(x)} f(t)dt =$ ________________

說明

$$\frac{d}{dx}\int_a^x f(t)dt = \lim_{h\to 0}\frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)dt}{h} = \lim_{h\to 0}\frac{\int_x^{x+h} f(t)dt}{h}$$

$$= \lim_{h\to 0}\frac{f(c)\cdot h}{h} \quad (\text{其中 } x\le c\le x+h\text{, by 積分均值定理})$$

$$= \lim_{h\to 0} f(c) = f(x) \quad (\text{因 } x\le c\le x+h \text{ 且 } f(x) \text{ 爲連續}) \quad \text{Q.E.D.}$$

例題 1.

Evaluate $\dfrac{d}{dx}\int_0^x \sqrt{t^2+1}\,dt$

解

例題 2. (精選範例 4-1)

Find the second derivative of $\int_{\pi/2}^{x^3} \cos t dt$

解

例題 3. (精選範例 4-2)

Let $G(x) = \int_0^x \left\{ s \int_0^s f(t)dt \right\} ds$, where $f(t)$ is continuous for all real t. Find $G(0)$, $G'(0)$, $G''(x)$, $G''(0)$ and $G'''(x)$

解